Phys 270 Final Exam

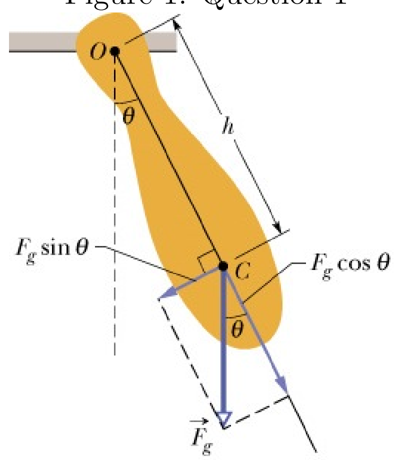
Time limit: 120 minutes

Each question worths 10 points.

Constants: $g = 9.8 m/s^2$, $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$.

1. (a) Figure 1 shows an object with moment of inertia I and mass m oscillating about the axis O. It is also given that the center of mass is at C, at a distance h from O, and you may assume $\theta \ll 1$ (small angle). Find the equation of motion for this oscillator in terms of θ and the given variables by applying Newton's second law (show the steps logically, no points will be given if the equation of motion is written without proof). From the equation, write down the angular frequency ω. (b) Suppose you are on an unknown planet, and you are trying to determine the value of g on that planet. Given $m = 2kg$, $h = 0.5m$, $I = 3kgm^2$, and with a stopwatch you measured the time it takes for the object to oscillate 50 times is $170s$. Find the value of g on the planet.

Figure 1: Question 1

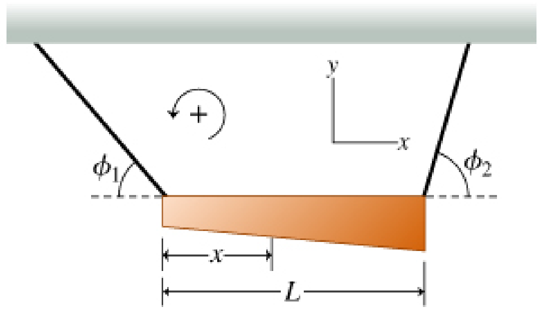


2. Consider a satellite of mass m orbiting around a planet of mass M. The radius of the orbit is r. (a) What is the period of the satellite in terms of the given variables (and other constants in physics)? [Show the steps in your calculations.] (b) What is its tangential speed v? (c) What is the kinetic energy of the satellite? (d) What is the ratio of KE/PE? You answer should be a pure number.

3. A non-uniform, horizontal bar of mass $m = 1.5kg$ is supported by two massless wires against gravity (Figure 2). The left wire makes an angle $\phi_1 = 30°$ with the horizontal, and the right wire makes an angle $\phi_2 = 50°$. The bar has length $L = 2m$. (a) Find the position of the center of mass of the bar, x, measured from the bar's left end. (b) Find the tensions (T_1, T_2) of the two wires.

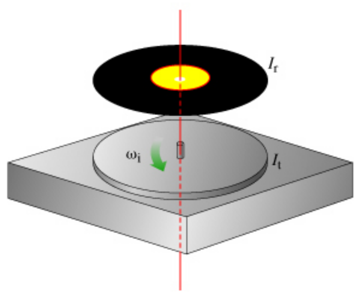
4. Figure 3 shows a turntable of moment of inertia I_t rotating at a constant angular velocity ω_i around an axis through the center and perpendicular to the plane of the disk (the disk's "primary axis of symmetry"). The axis of the disk is vertical and the disk is supported by frictionless bearings. The motor of the turntable is off, so there is no external torque being applied to the axis. Another disk (a record) is dropped onto the first such that it lands coaxially (the axes coincide). The moment of inertia of the record is I_r. The initial angular velocity of the second disk is zero. There is friction between the two disks. After this rotational collision, the disks will eventually rotate with the same angular velocity. (a) What is the final angular velocity, ω_f, of the two disks? [Express

Figure 2: Question 3



your answer for (a) in terms of I_t, I_r, and ω_i.] (b) Find the ratio of the rotational kinetic energy KE_f/KE_i. [Express your answer for (b) in terms of I_t, I_r, but not in ω_i.]

Figure 3: Question 4



5. Two particles move perpendicular to each other until they collide. Particle 1 has mass m and momentum of magnitude $2p$, and particle 2 has mass $2m$ and momentum of magnitude p. Note: Magnitudes are not drawn to scale in any of the figures. (a) Suppose that after the collision, the particles "trade" their momenta, as shown in Figure 4 . That is, particle 1 now has magnitude of momentum p, and particle 2 has magnitude of momentum $2p$; furthermore, each particle is now moving in the direction in which the other had been moving. What is the change in kinetic energy ΔKE in the collision? (b) Consider an alternative situation: This time the particles collide completely inelastically (so the final momentum is no longer what is indicated in the figure). What is the change in kinetic energy ΔKE in this case?

6. Figure 5 shows the bottom of an elevator shaft. A spring is installed at the bottom to stop the elevator before it hits the ground should an accident occur. The mass of the elevator is $2000kg$, and a friction of $17000N$ is applied to slow down the elevator during a fall. Assuming the elevator is moving at a speed of $v_1 = 4m/s$ when it hits the spring, what should k be in order to stop the elevator in $2m$?

7. Figure 6 shows block 1, of mass $m_1 = 10kg$ connected over a pulley on an incline with angle $\alpha = 30°$ to block 2, of mass $m_2 = 20kg$. The two masses move a distance of $s = 5m$. The coefficient of friction between the incline and m_1 is $\mu = 0.2$. You may assume m_1 is moving up the incline. (a)

Figure 4: Question 5

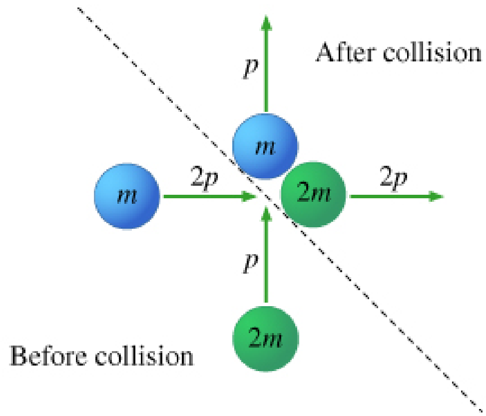
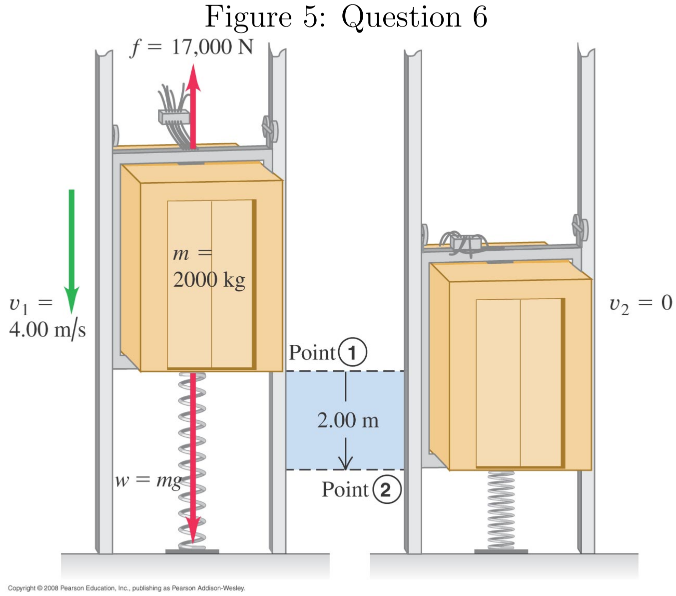
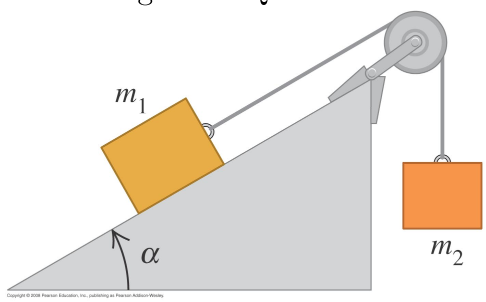


Figure 5: Question 6



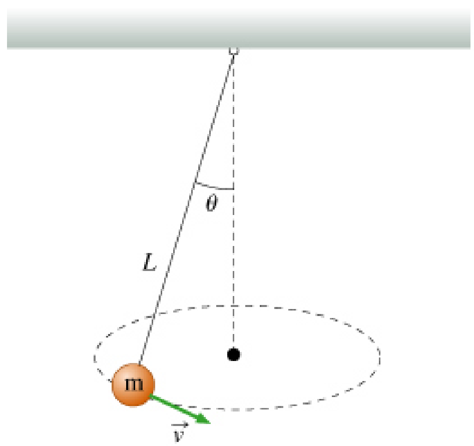
Draw the force diagrams. (b) Find the force of friction on m_1. (c) Find the acceleration and the tension. (d) What is the work done on m_1 by gravity?

Figure 6: Question 7



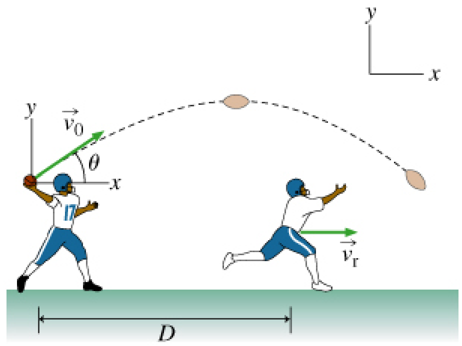
8. Figure 7 shows a bob of mass m is suspended from a fixed point with a massless string of length L (i.e., it is a pendulum). You are to investigate the motion in which the string moves in a cone with half-angle θ. Express your answers below in terms of some or all of the variables m, L, and θ, as well as the acceleration due to gravity g. (a) What tangential speed, v, must the bob have so that it moves in a horizontal circle with the string always making an angle θ from the vertical? (b) How long does it take the bob to make one full revolution (one complete trip around the circle)?

Figure 7: Question 8



9. Figure 8 shows quarterback is set up to throw the football to a receiver who is running with a constant velocity $\vec{v}_r$ directly away from the quarterback and is now a distance D away from the quarterback. The quarterback figures that the ball must be thrown at an angle θ to the horizontal and he estimates that the receiver must catch the ball a time interval t_c after it is thrown to avoid having opposition players prevent the receiver from making the catch. In the following you may assume that the ball is thrown and caught at the same height above the level playing field. Assume that the y coordinate of the ball at the instant it is thrown or caught is $y = 0$ and that the horizontal position of the quarterback is $x = 0$. Express all your answers below for in terms of g, D, t_c, and v_r (but <u>not</u> θ and v_0). (a) Find v_{0y}, the vertical component of the velocity of the ball when the quarterback releases it. (b) Find v_{0x}, the initial horizontal component of velocity of the ball. (c) Find the speed v_0 with which the quarterback must throw the ball. (d) Find the angle θ above the horizontal at which he should throw it.

Figure 8: Question 9



10. (a) Prove the law of conservation of momentum using Newton's Laws of Motion. (b) Prove that $PE_G = -\frac{GMm}{r}$ implies mgh on the surface of the Earth. (c) Vector $\vec{v}$ has magnitude 30.0 and is directed 70.0° counterclockwise from the positive y axis (NOT the x axis!). Express it in unit-vector notation (i.e. with $\hat{\imath}, \hat{\jmath}$).